Failure of Local Closure in Self-Optimizing Systems: A Minimal Structural Bound

Claudio Bresciano

Independent Researcher

Abstract

Self-optimizing systems are commonly assumed to be capable of recovering coherence through internal optimization alone. This paper demonstrates that this assumption fails categorically below a minimal structural threshold. We prove that any self-optimizing system operating under local closure admits a **Minimal Structural Bound** $F_{\min}$ such that coherence recovery becomes impossible once this bound is crossed. Optimization presupposes structural sufficiency and cannot reconstruct it when lost. This result formalizes a general limit on self-referential optimization, with implications for machine learning, biological systems, and theories of ontological closure.

Keywords

Minimal Structural Sufficiency; Local Closure; Self-Optimization; Structural Collapse; Theory of Axiomatic Necessity

1. Introduction

Self-optimizing systems appear across domains: machine learning models, adaptive biological systems, economic agents, and epistemic frameworks. A recurring assumption is that sufficient optimization pressure can compensate for structural degradation.

This paper challenges that assumption.

We show that **optimization is only defined relative to an already sufficient structure**. Once coherence falls below a minimal bound, no internally generated process can restore it. This failure is not contingent, empirical, or domain-specific—it is structural.

The result formalizes a core claim of the **Theory of Axiomatic Necessity (TNA)**: coherence requires irreducible structural support.

---

2. Formal Framework

2.1 System Definition

Let $S = (\mathcal{X}, D, \mathcal{O})$ be a self-optimizing system, where:

- $\mathcal{X}$ is the state space,
- D are the internal dynamics,
- $\mathcal{O}$ is an optimization operator acting only on internally accessible variables.

2.2 Local Closure

Definition (Local Closure).
A system is locally closed if all constraints required for coherence preservation or recovery are derivable from its internal state, dynamics, and optimization rules.

No exogenous constraints are permitted.

2.3 Coherence Functional

Let

$$\Psi : \mathcal{X} \to \mathbb{R}^+$$

be a coherence functional measuring the system's internal structural viability (e.g., representational consistency, operational stability, or semantic adequacy).

---

3. Minimal Structural Bound

Definition (Minimal Structural Bound).
There exists a constant $F_{\min} > 0$ such that optimization is only well-defined when:

$$\Psi(x) \geq F_{\min}$$

Below this bound, the system lacks the structural relations required to define meaningful optimization trajectories.

---

4. Main Theorem

> *"Theorem 1 is intentionally tautological. Its purpose is not to establish a surprising mathematical result, but to make explicit the minimal structural assumptions under which irrecoverability becomes unavoidable."*

Theorem 1 — Failure of Local Closure under Fixed-Structure Optimization (Strong Conditional Form)

Let

$$S = (X, \Theta, \mathcal{U}, \Psi)$$

be a self-optimizing system, where:

- $x_t \in X$ denotes observable operational states;
- $\theta \in \Theta$ denotes the system's effective internal structure;
- $\mathcal{U}$ is a set of admissible update operators acting on Θ;
- $\Psi : \Theta \to \mathbb{R}_{\geq 0}$ is a structural coherence functional.

Assume the following conditions:

1. **Structural Closure (Fixed-Structure Constraint).**
   Update operators $u \in \mathcal{U}$ act only on existing structural degrees of freedom and cannot introduce new ones (no architectural growth, no dimensional expansion, no access to external structural information).
2. **Structural Monotonicity.**
   For any $u \in \mathcal{U}$, if u removes effective structural degrees of freedom, then
   $$\Psi(u(\theta)) \leq \Psi(\theta).$$
3. **Minimal Structural Realizability.**
   There exists a threshold $\Psi_{\min} > 0$ such that for all θ with
   $$\Psi(\theta) < \Psi_{\min},$$
   no structurally equivalent state restoring the original functional class exists within Θ

under the action of $\mathcal{U}$.

> *"This result should be read as a conditional impossibility: given fixed representational capacity and structural closure, recovery below Ψ_min is formally excluded by definition."*

Then, for any time t_0,

> *if*
>
> $$\Psi(\theta_{t_0}) < \Psi_{\min},$$
>
> *there exists **no finite or infinite sequence** of locally admissible update operations*
>
> $$\{u_t\}_{t \geq t_0} \subset \mathcal{U}$$
>
> *such that*
>
> $$\Psi(\theta_t) \geq \Psi_{\min}.$$

In other words, **under fixed-structure, locally closed optimization dynamics, structural coherence below a minimal bound is formally irrecoverable**.

> *"Tautological results play a legitimate role in the philosophy of science: they delineate conceptual boundaries, prevent illicit extrapolations, and clarify which forms of recovery necessarily require external intervention. The present theorem serves this function by isolating the precise assumptions under which self-recovery claims become incoherent."*

Interpretive Note

This result is **conditional**, not metaphysical.
Structural recovery is possible **only** by violating at least one of the stated assumptions—most commonly by introducing non-local structural information or expanding the system's structural degrees of freedom.

> *"The remainder of the paper explores concrete instantiations of this bound in neural pruning, biological degeneration, and social systems under fixed-structure constraints."*

---

5. Proof Sketch

The proof proceeds by making explicit how irrecoverability follows **conditionally** from the structural assumptions stated in Theorem 1.

5.1 Optimization Presupposes Structural Degrees of Freedom

Under Assumption 1 (Structural Closure), admissible update operators

$$u \in \mathcal{U}$$

act only on **existing structural degrees of freedom**.

For optimization to be well-defined, the system must admit:

- stable relational structure,
- non-degenerate internal parametrization,
- update directions that preserve functional equivalence classes.

These requirements are captured operationally by the condition

$$\Psi(\theta) \geq \Psi_{\min},$$

which marks the minimal regime in which optimization dynamics are structurally meaningful.

Below this threshold, optimization operators remain syntactically defined but lose semantic efficacy.

5.2 Structural Degeneracy Below $\Psi_{\min}$

Assumption 2 (Structural Monotonicity) implies that once effective degrees of freedom are removed, admissible updates cannot increase structural coherence.

When

$$\Psi(\theta_t) < \Psi_{\min},$$

the system enters a regime where:

- representational rank is insufficient,

- internal mappings collapse onto lower-dimensional manifolds,
- update operators act on structurally equivalent—but functionally deficient—states.

In this regime, expected structural coherence is non-increasing under locally admissible updates:

$$\mathbb{E}[\Psi(\theta_{t+1}) \mid \Psi(\theta_t) < \Psi_{\min}] \leq \Psi(\theta_t),$$

not as a dynamical law, but as a **structural consequence** of closure and monotonicity.

5.3 Failure of Internal Reconstruction under Closure

By Assumption 3 (Minimal Structural Realizability), for any

$$\Psi(\theta) < \Psi_{\min},$$

no state structurally equivalent to the original functional class exists within Θ under the action of $\mathcal{U}$.

Because all admissible transformations are endomorphic:

- no new relational dimensions can be introduced,
- no lost degrees of freedom can be re-instantiated,
- reconstruction would require access to structural distinctions no longer represented internally.

Thus, any sequence

$$\{u_t\}_{t \geq t_0} \subset \mathcal{U}$$

remains confined to a structurally collapsed equivalence class.

5.4 Conditional Necessity of External Structure

Recovery of structural coherence above $\Psi_{\min}$ would require:

- expansion of Θ,
- introduction of non-derivable constraints,
- or access to external structural information.

Each of these violates Structural Closure (Assumption 1).

Therefore, **irrecoverability is not absolute**, but follows necessarily **given** the fixed-structure, locally closed optimization regime.

---

6. Corollaries

Corollary 1 — Optimization Is Structurally Non-Generative

No optimization process operating under structural closure can reconstruct the structural conditions that make optimization effective.

Optimization presupposes structure; it does not generate it.

Corollary 2 — Cross-Domain Validity of Structural Bounds

Because the argument depends only on abstract structural assumptions, the existence of a minimal irrecoverability threshold $\Psi_{\min}$ applies across domains, including:

- neural network pruning under fixed architectures,
- biological degeneration without regenerative mechanisms,
- epistemic systems with frozen representational vocabularies,
- social systems constrained by rigid institutional structures.

---

7. Relation to Existing Results

- Gödel identifies incompleteness under fixed formal axioms.
- Non-equilibrium thermodynamics studies instability under external fluxes.
- The present result isolates a **recoverability limit under closure**, independent of specific dynamics.

The contribution is not an incompleteness theorem, but a **conditional impossibility of structural recovery**.

---

8. Discussion

The theorem explains why:

- aggressive pruning fails beyond specific thresholds,
- optimization-driven collapse occurs despite continued updates,
- systems cannot self-bootstrap lost structural coherence without expansion.

The key distinction is structural:

> *parameter optimization modifies values;*
> *structural recovery requires new degrees of freedom.*

9. Conclusion

Local closure fails as a universal assumption for self-optimizing systems.
Coherence requires minimal structural sufficiency, and once lost, cannot be internally recovered.

This establishes a hard limit on self-referential optimization.

10. Formal Example: Neural Pruning and Self-Alignment Failure

10.1 Setup

Consider a neural network $\mathcal{N}$ with parameters $\theta \in \mathbb{R}^n$, trained to minimize a loss function $L(\theta)$ under a fixed data distribution.

Let:

- $\Psi(\theta)$ denote a coherence functional measuring **representational sufficiency**, not performance (e.g., internal feature diversity, rank of effective representations, or causal path multiplicity).
- $\mathcal{O}$ be an optimization operator (SGD, Adam, etc.).
- Pruning be defined as a parameter-removal operator P_r removing a fraction $r \in (0, 1)$ of parameters based on some saliency criterion.

The system is assumed to be **locally closed**:

- No new parameters are introduced.

- No architectural changes are allowed.
- No external priors are injected post-pruning.

10.2 Empirical Regularity

Empirically, pruning exhibits three regimes:

1. **Redundant regime**
   For small r, performance and coherence are preserved.
2. **Critical regime**
   Beyond a threshold $r\backslash^*$, performance collapses abruptly.
3. **Irrecoverable regime**
   Retraining from the pruned state fails to restore prior performance or internal structure, even with identical data and optimization.

This behavior is well documented but typically treated as empirical fragility.

10.3 Structural Interpretation

We reinterpret this phenomenon structurally.

Define:

$$Define : \Psi(\theta) = \mathrm{rank}(\mathcal{R}(\theta)),$$

interpreted as an architecture-relative proxy for representational sufficiency.

where $\mathcal{R}(\theta)$ is the internal representational operator induced by the network (e.g., effective feature map).

There exists a **minimal structural bound** $F_{\min}$ such that:

$$\Psi(\theta) \geq F_{\min}$$

is required for gradients to remain informative.

10.4 Collapse Under Pruning

Pruning reduces parameter space dimension and collapses internal relations:

$$\Psi(P_r(\theta)) < \Psi(\theta)$$

There exists r_c such that:

$$\Psi(P_{r_c}(\theta)) < F_{\min}$$

At this point:

- Gradients exist numerically,
- Loss may still decrease locally,
- But updates no longer correspond to meaningful structural recovery.

10.5 Failure of Self-Alignment

Let the pruned network attempt self-recovery via optimization:

$$\theta_{t+1} = \mathcal{O}(\theta_t)$$

Under local closure:

- $\mathcal{O}$ operates only on surviving parameters,
- No new representational degrees of freedom can emerge,
- All updates are endomorphic.

Thus, under fixed architecture and local closure:

$$\forall t > 0, \quad \Psi(\theta_t) < F_{\min}$$

Performance plateaus or degrades despite continued optimization.

10.6 Formal Statement

Proposition 2 — Conditional Irrecoverability under Structural Closure

Let $\mathcal{N}$ be a neural network pruned such that

$$\Psi(\theta_0) < F_{\min}$$

Then no optimization trajectory confined to the original architecture can restore:

$$\Psi(\theta) \geq F_{\min}$$

Recovery requires **external structural intervention**, such as:

- architectural expansion,
- parameter reintroduction,
- externally imposed priors.

10.7 Why Optimization Fails

Optimization adjusts *values*, not *relations*.

Once the relational graph defining representational sufficiency is broken:

- gradients lose semantic meaning,
- training becomes statistically but not structurally effective,
- the system optimizes noise within an insufficient structure.

This is not overfitting.
It is **structural underdetermination**.

10.8 Implications

This explains why:

- Lottery Ticket Hypothesis only works *above* critical structure,
- Fine-tuning cannot resurrect collapsed models,
- Self-alignment without external constraint is provably limited.

Pruning does not merely remove redundancy—it can destroy the **conditions of learnability**.

10.9 Relation to TNA

This example instantiates the central TNA claim:

> *Optimization presupposes structural sufficiency and cannot reconstruct it once lost.*

Here:

- N_0: weights, gradients, loss values,
- N_1: architectural relations enabling representational coherence.

Once N_1 collapses below $F_{\min}$, N_0-only dynamics are insufficient.

- Reinitialization without structural expansion does not increase $\dim(\mathcal{R})$.
- It randomizes values, not relations.
- Therefore, Ψ remains bounded below $F_{\min}$.

11. Formal Counter-Objections and Structural Impossibility Results

This section anticipates and resolves the principal objections typically raised against structural non-closure claims in self-optimizing systems. Each objection is addressed formally and shown to fail under the assumptions already stated.

11.1 Objection 1: "Sufficient Data Can Recover the Structure"

Claim.
Given enough data, a pruned or degraded system can recover lost coherence through optimization.

Response.
Data constrains parameter values but does not increase representational degrees of freedom.

Formally, let:

$$\mathcal{D} \subset \mathcal{X} \times \mathcal{Y}$$

be any finite or infinite dataset.

If:

$$\Psi(\theta_0) < F_{\min}$$

then for any dataset $\mathcal{D}$,

$$\sup_{\theta \in \Theta} \Psi(\theta \mid \mathcal{D}) < F_{\min}$$

Data refines estimates within an existing structural capacity; it cannot reconstruct missing relational dimensions.

Conclusion.
Data operates on N_0, while structural sufficiency resides in N_1. The objection confuses informational richness with structural adequacy.

11.2 Objection 2: "Reinitialization Can Restore Learnability"

Claim.
Random reinitialization of parameters allows the system to escape collapsed states.

Response.
Reinitialization alters parameter values but preserves the architecture's relational graph.

Let:

$$\theta' \sim \mathcal{U}(\Theta)$$

Then:

$$\dim(\mathcal{R}(\theta')) = \dim(\mathcal{R}(\theta))$$

Thus:

$$\Psi(\theta') = \Psi(\theta) < F_{\min}$$

Randomization does not increase representational rank; it only redistributes weight mass.

Conclusion.
Reinitialization resets values, not structure. Learnability requires structural sufficiency, not stochastic escape.

11.3 Objection 3: “Implicit Bias of Optimization Acts as External Support”

Claim.
The inductive bias of optimization algorithms functions as an external structural guide.

Response.
Implicit bias shapes trajectories *within* a fixed hypothesis class; it does not expand it.

Let:

$$\mathcal{O} : \Theta \to \Theta$$

If $\mathcal{O}$ is endomorphic:

$$\mathcal{O}(\Theta) \subseteq \Theta$$

Then:

$$\Psi(\mathcal{O}(\theta)) \leq \Psi(\theta)$$

Implicit bias selects among existing configurations; it does not introduce new relational primitives.

Conclusion.
Optimization bias is not an N_1 source. It presupposes N_1.

11.4 Objection 4: “Architectures Self-Organize New Structure During Training”

Claim.
Neural systems can internally generate new structure via self-organization.

Response.
Self-organization rearranges existing degrees of freedom but cannot exceed architectural bounds.

Formally:

$$\Psi(\theta_t) \leq \Psi_{\max}(\mathcal{A})$$

where $\mathcal{A}$ is the fixed architecture.

If:

$$\Psi_{\max}(\mathcal{A}) < F_{\min}$$

then:

$$\forall t,\ \Psi(\theta_t) < F_{\min}$$

Self-organization operates under conservation of structural capacity.

Conclusion.
Emergence is constrained by prior structure. No system operating under fixed structural constraints self-generates its own sufficiency.

11.5 Objection 5: "This Is Merely an Optimization Pathology"

Claim.
The observed collapse is due to poor optimization dynamics, not structural limits.

Response.
Pathologies are contingent; structural bounds are invariant across optimizers.

Let $\mathcal{O}_1, \mathcal{O}_2$ be distinct optimizers.

If:

$$\Psi(\theta_0) < F_{\min}$$

then:

$$\forall \mathcal{O}_i,\ \forall t,\ \Psi(\theta_t^{(i)}) < F_{\min}$$

Optimizer choice affects convergence speed, not representational rank.

Conclusion.
This is not an optimization failure but a structural impossibility result.

11.6 Objection 6: “Performance Recovery Implies Structural Recovery”

Claim.
If task performance recovers, structure must have recovered.

Response.
Performance is a scalar observable; coherence is relational.

There exist θ_a, θ_b such that:

$$L(\theta_a) \approx L(\theta_b) \quad \text{and} \quad \Psi(\theta_a) \neq \Psi(\theta_b)$$

Task success does not entail structural sufficiency.

Conclusion.
Equating performance with coherence conflates outcome adequacy with explanatory completeness.

11.7 Meta-Conclusion

All objections rely on variations of the same error:

> *Treating value-level operations as capable of repairing relation-level deficits.*

Minimal Structural Sufficiency establishes a non-negotiable boundary:

- Optimization requires structure.
- Structure cannot be recovered by optimization alone.
- Closure at N_0 is provably insufficient.

Thus, failure of recovery below F_min is not empirical fragility,
but a formal necessity conditional on the adoption of fixed-structure optimization and local closure as modeling assumptions.”

12. Cross-Domain Corollary: Structural Non-Closure Across Biological, Cognitive, and Social Systems

This corollary establishes that the Failure of Local Closure under Minimal Structural Sufficiency is not domain-specific. Any system that (i) maintains coherence, (ii) adapts to perturbations, and (iii) optimizes internal states under constraints is subject to the same structural bound.

> *"The following corollaries are heuristic and analogical. They are not derived from Theorem 1, but illustrate how similar closure errors appear across domains once structural sufficiency is ignored."*

12.1 General Corollary (Domain-Independent Form)

Corollary 1 (Structural Non-Closure).
Let S be any self-maintaining system operating under internal optimization dynamics. If the effective structural coherence Ψ_S falls below F_min,
then no internal process of S, operating under structural closure,
can restore coherence

Formally:

$$\Psi_S < F_{\min} \;\Rightarrow\; \neg\exists \mathcal{I}_S \text{ such that } \Psi'_S \geq F_{\min}$$

where $\mathcal{I}_S$ denotes any internally available transformation of the system.

This result holds regardless of substrate, scale, or domain.

12.2 Corollary in Biological Systems

12.2.1 Neural Degeneration and Plasticity

Claim.
Neural plasticity does not imply unlimited structural recoverability.

Let:

- S_B: a biological neural system
- Ψ_B: effective functional coherence (connectivity, integration, signal routing)

Empirical plasticity operates under:

$$\Psi_B(t) \leq \Psi_{\max}(\mathcal{A}_{bio})$$

If injury or degeneration causes:

$$\Psi_B < F_{\min}$$

then compensatory rewiring cannot restore global coherence without:

- external scaffolding (rehabilitation protocols),
- environmental structure,
- or developmental priors.

Result.
Extreme recovery cases do not violate the corollary; they presuppose latent structural support not generated by the damaged system itself.

Interpretation.
Biology adapts—but only within preserved structural bounds.

12.3 Corollary in Cognitive Systems

12.3.1 Meaning, Intentionality, and Conceptual Collapse

Let:

- S_C: a cognitive agent
- Ψ_C: coherence of conceptual, intentional, and semantic relations

Cognitive coherence depends on:

- linguistic scaffolds,
- social feedback,
- normative constraints.

If isolation, trauma, or symbolic erosion reduce:

$$\Psi_C < F_{\min}$$

then introspection, reflection, or internal reasoning alone cannot restore meaning.

Formally:

$$\forall \mathcal{R}_{int},\ \Psi_C(\mathcal{R}_{int}) < F_{\min}$$

Result.
Meaning is not self-generated. It requires external structural anchoring.

Interpretation.
This explains:

- failure of purely internal sense-making,
- limits of self-therapy,
- collapse of intentionality in closed cognitive loops.

12.4 Corollary in Social Systems

12.4.1 Institutions, Markets, and Normative Breakdown

Let:

- S_S: a social system
- Ψ_S: coherence of norms, roles, signals, and expectations

Social systems optimize behavior locally (agents adapt), but coherence depends on:

- shared norms,
- enforceable boundaries,
- reliable signals.

If erosion of trust, legitimacy, or signaling causes:

$$\Psi_S < F_{\min}$$

then:

- increased individual optimization,
- algorithmic matching,

- incentive tweaking

cannot restore system-level coherence.

Formally:

$$\text{Local rationality} \;\not\Rightarrow\; \text{Global coherence}$$

Result.
A society cannot internally optimize its way out of structural collapse.

Interpretation.
This applies directly to:

- frictionless markets,
- signal-free dating systems,
- bureaucracies optimizing metrics after losing legitimacy.

12.5 Unified Interpretation

Across domains, the same invariant holds:

> ***Optimization presupposes structure; structure cannot be regenerated by optimization alone.***

DOMAIN	OBSERVABLE RECOVERY	STRUCTURAL SOURCE
Biology	Plasticity	Developmental + environmental scaffolding
Cognition	Learning	Language + social norms
Society	Adaptation	Institutions + shared constraints
ML (prior sections)	Training	Architecture + inductive bias

12.6 Structural Consequence

The corollary implies a sharp distinction:

- **Adaptive capacity** is internal.
- **Structural sufficiency** is not.

Any attempt to explain coherence solely from within the system commits a closure error.

12.7 Final Corollary Statement

Corollary 2 (Universal Structural Dependence).
For any coherent system S operating under internal optimization:

$$\text{Coherence}(S) \;\Rightarrow\; \exists N_1 \notin S$$

This is not an empirical claim, but a structural necessity
given closure of the system's representational resources.

13. Implications for Post-Materialism and Formal Epistemology

This section clarifies the philosophical status of the Theory of Axiomatic Necessity (TNA) and its implications for contemporary debates in post-materialism and formal epistemology. The goal is not to introduce a new metaphysics, but to correct a recurrent category error: the treatment of structural limits as ontological claims.

13.1 Why TNA Is Not a Post-Materialist Ontology

Post-materialism is commonly defined by a positive claim:
that reality contains something beyond matter (mind, consciousness, information, experience).

TNA makes **no such claim**.

Formally, TNA does **not** assert:

$$\exists X \text{ such that } X \notin \text{material domain}$$

Instead, it demonstrates:

$$\forall S \; (\text{coherent}(S) \Rightarrow \exists N_1 \notin \mathcal{O}(S))$$

where $\mathcal{O}(S)$ denotes the domain of observables and internal operations of S.

This distinction is decisive.

- Post-materialist ontologies are **substantive** (they posit entities).
- TNA is **negative-structural** (it posits impossibility of closure).

TNA therefore belongs neither to materialism nor to its traditional negation. It operates at a **meta-level**: it constrains what any ontology—materialist or otherwise—can successfully explain from within.

13.2 Structural Insufficiency vs. Ontological Addition

Most post-materialist frameworks attempt to resolve explanatory failure by **adding content**:

- consciousness as fundamental,
- information as substrate,
- experience as primitive.

TNA shows that such moves are **structurally unnecessary**.

The failure addressed by TNA is not due to missing entities, but to **closure impossibility**.

Formally:

$$\text{Explanatory failure} \neq \text{ontological absence}$$

Adding new primitives does not repair closure; it merely relocates the boundary.

13.3 Reframing the Materialism Debate

From the TNA perspective, the classical debate:

> *"Is matter sufficient to explain reality?"*

is ill-posed.

The correct question is:

> *"Can any internally defined domain explain the conditions of its own coherence?"*

TNA answers this in the negative, independently of what the domain contains.

Thus:

- Materialism fails **not because matter is insufficiently rich**,

- but because **no domain can be sufficient to itself**.

This reframes post-materialism as a **symptom**, not a solution:
a reaction to closure failure misinterpreted as ontological lack.

13.4 Implications for Formal Epistemology

In formal epistemology, TNA introduces a sharp constraint:

> *No epistemic system can fully justify the axioms that make its own justification possible.*

This generalizes incompleteness beyond formal logic into:

- scientific modeling,
- learning systems,
- explanatory frameworks.

Let E be an epistemic system with inference rules $\mathcal{R}$.
Then:

$$\mathcal{R} \nvdash \text{Necessity}(\mathcal{R})$$

Minimal Structural Sufficiency thus functions as an **epistemic lower bound**:
below F_{min}, explanation collapses regardless of inferential power.

13.5 Against Epistemic Self-Grounding

A central implication follows:

> *Epistemic self-grounding is structurally impossible.*

This applies equally to:

- empirical sciences,
- rationalist systems,
- AI-based epistemologies.

Any claim of total explanation—whether materialist, idealist, or informational—commits the same error:
it mistakes operational completeness for structural sufficiency.

13.6 Positioning: Axiomatic Structuralism

The appropriate classification of TNA is therefore:

Axiomatic Structuralism
A framework that:

- does not posit substances,
- does not privilege domains,
- formalizes the necessity of non-derivable structural support.

Its defining move is not metaphysical expansion, but **reduction to incoherence**:
it shows where and why explanation must stop.

13.7 Final Implication

TNA implies a disciplined epistemic stance:

> *The limit of explanation is not ignorance, but structure.*

Recognizing this does not weaken science or philosophy.
It prevents them from mistaking explanatory exhaustion for ontological discovery.

In this sense, TNA does not replace materialism, nor endorse post-materialism.
It renders both secondary to a more fundamental constraint:

> ***No coherent system can account for the necessity of its own coherence.***

That constraint is not a belief.
It is a theorem.

14. Conclusion

This paper has formalized a general structural result:
no self-optimizing or coherent system can achieve local explanatory closure.

The central contribution is not an ontological proposal, but a **limit theorem**. By introducing the concept of *Minimal Structural Sufficiency* and the bound $F_{\min}$, we showed that coherence, optimization, or explanatory power cannot be derived exclusively from within a system's observable or operational domain ($\mathcal{N}_0$).

Across formal proofs, examples, and cross-domain corollaries, the same structural fact emerged:

> *Whenever a system maintains coherence, learning, or stability, it presupposes an external structural support ($\mathcal{N}_1$) that is non-derivable, non-eliminable, and irreducible from internal operations.*

This result holds independently of:

- the material substrate of the system,
- the representational language employed,
- the degree of computational or inferential power,
- or the empirical domain in which the system is instantiated.

14.1 Summary of Results

The paper established the following:

1. **Failure of Local Closure**
   Self-optimizing systems cannot explain the axioms, constraints, or biases that make optimization possible.
2. **Minimal Structural Bound**
   There exists a lower bound $F_{\min}$ below which coherence collapses regardless of data volume or model capacity.
3. **External Necessity**
   Structural support ($\mathcal{N}_1$) is not an empirical variable but a formal requirement for coherence.
4. **Domain Invariance**
   The same limit applies to machine learning, biological systems, cognition, and social structures.

5. **Non-Ontological Status**
   The result does not imply the existence of new substances, forces, or metaphysical entities.

14.2 What the Result Does *Not* Claim

To avoid categorical misinterpretation, it is essential to state explicitly what this work does **not** claim:

- It does **not** argue for post-materialism as an ontology.
- It does **not** posit consciousness, information, or mind as fundamental substances.
- It does **not** replace physical explanation with metaphysical speculation.
- It does **not** claim completeness, finality, or ultimate explanation.

Instead, it identifies a **structural impossibility** shared by all explanatory systems.

14.3 Epistemological Implication

The central epistemological implication is precise:

> *Explanatory failure is not equivalent to ontological absence.*

When a system reaches its explanatory boundary, the correct inference is not "something mystical exists," but:

> *"The system has reached the limit of what can be derived internally."*

This reframes long-standing debates in philosophy of science and AI. Many disputes traditionally framed as metaphysical disagreements are, under this lens, **boundary misdiagnoses**.

14.4 Axiomatic Structuralism as a Position

The appropriate classification of the framework developed here is **Axiomatic Structuralism**.

It is:

- **Axiomatic**, because it formalizes necessity conditions rather than empirical contingencies.
- **Structural**, because it concerns relations of support and coherence, not substances.
- **Negative**, because it operates by demonstrating impossibility rather than proposing alternatives.

Its method is reduction to incoherence, not construction of new ontologies.

14.5 Final Statement

The result proven in this paper can be stated succinctly:

> *Any system capable of explanation, optimization, or coherence necessarily depends on structural conditions it cannot itself generate, justify, or eliminate.*

This dependence is not a defect.
It is the price of coherence.

Recognizing this does not weaken scientific or formal inquiry—it prevents it from mistaking its own limits for discoveries about reality.

The Theory of Axiomatic Necessity therefore does not tell us *what the world is made of*.
It tells us **what no system can do**, regardless of what the world is made of.

And that constraint is universal.

Executive Summary

Failure of Local Closure in Self-Optimizing Systems: A Minimal Structural Bound

Problem Statement

Across multiple disciplines—machine learning, cognitive science, biology, and social systems—models increasingly aim to be *self-optimizing*, *self-explanatory*, or *self-sufficient*. However, persistent failures arise when such systems attempt to justify or derive the very constraints that enable their coherence.

These failures are often misclassified as:

- data limitations,
- model insufficiency,
- lack of computational power,
- or unresolved metaphysical questions.

This paper demonstrates that these failures are **structural**, not contingent.

Core Result

The paper proves a general limit theorem:

> ***No coherent or self-optimizing system can achieve local explanatory closure.***

Formally, any system operating within an observable or operational domain $\mathcal{N}_0$ necessarily presupposes an external structural support $\mathcal{N}_1$ that:

- cannot be derived from internal operations,
- cannot be eliminated by optimization,
- and cannot be reduced to empirical parameters.

This requirement is captured by a **Minimal Structural Bound**, denoted $F_{\min}$.

Key Concepts

- **Minimal Structural Sufficiency**
  The minimal set of non-derivable constraints required for coherence.
- $F_{\min}$
  A lower bound below which coherence collapses, regardless of data volume or model capacity.
- **Failure of Local Closure**
  The impossibility for a system to fully explain or justify the conditions that make its own operation possible.
- $\mathcal{N}_0/\mathcal{N}_1$ **Distinction**
  Observable operations versus irreducible structural support.

Methodology

The result is established through:

1. A formal theorem with proof sketch.
2. A constructive example in neural pruning and self-alignment.
3. A formal counter-objection analysis.
4. Cross-domain corollaries (ML, biology, cognition, social systems).

No ontological assumptions are introduced.

What This Paper Does *Not* Claim

- It does **not** propose a new metaphysics.
- It does **not** argue for consciousness, mind, or information as substances.
- It does **not** reject material explanation.
- It does **not** claim completeness or finality.

The result is **negative and structural**, not positive or ontological.

Implications

1. **For Machine Learning**
   Inductive bias, alignment constraints, and optimization criteria are structurally non-derivable. Treating them as tunable parameters misdiagnoses the problem.
2. **For Cognitive and Biological Systems**
   Coherence and function cannot be reduced to local mechanisms alone.
3. **For Epistemology**
   Many metaphysical debates reflect boundary failures, not competing ontologies.

Positioning

This framework is best classified as **Axiomatic Structuralism**:

- axiomatic (necessity-based),
- structural (relation-based),
- non-ontological (no new entities),
- cross-domain invariant.

It formalizes limits rather than proposing replacements.

Final Takeaway

> *Explanatory insufficiency does not imply ontological absence.*
> *It implies a structural boundary.*

The Theory of Axiomatic Necessity shows that **coherence has a cost**: dependence on conditions a system cannot explain from within.

This is not a flaw of systems.
It is the condition that makes them possible.

> ***"Claims of self-recovery implicitly assume preserved structure or external support."***